Conference Paper

Free Vibration Analysis of The Moderately Thick Laminated Composite Rectangular Plate on Two-Parameter Elastic Foundation with Elastic Boundary Conditions

H. Zhang, D.Y. Shi, and Q.S. Wang

College of Mechanical and Electrical Engineering, Harbin Engineering University, 150001 Harbin, China

Corresponding Author: D.Y. Shi; email: shidongyan@hrbeu.edu.cn

Abstract

An improved Fourier series method is presented for the free vibration analysis of the moderately thick laminated composite rectangular plate with general elastic supports and point supports resting on an elastic foundation. The approach is based on the first order shear deformation theory and foundation effect using two-parameter Pasternak foundation model. The displacement and rotation functions are generally sought, regardless of boundary conditions, as Fourier series and supplementary functions. All the series expansion coefficients are determined using the Rayleigh-Ritz technique. The excellent accuracy of the current results is validated by comparing them with existing results.

Keywords: improved Fourier series, composite rectangular plate, elastic supports, two-parameter Pasternak foundation, Rayleigh-Ritz technique

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1. Introduction

The laminated composite plates, as important structural components, are often supported by elastic foundations. Therefore, it is necessary to understand the vibration of these laminated plates. Shen [1] carried out the postbuckling analysis of simply supported composite laminated plates on Pasternak-type elastic foundation. Xiang et al. [2] analyzed the vibration of moderately thick simply supported rectangular laminates on Pasternak foundation. Huang et al. [3] presented a finite strip method for a three-span simply supported plate resting on elastic foundations. A lot of research has been done for the plates resting on foundations, but the study of the dependence of the vibration for the moderately thick laminated composite plate on the boundary conditions is relatively few.

This paper presents an improved Fourier series method for the free vibration analysis of the moderately thick laminated plate with different boundary conditions on Pasternak foundations.

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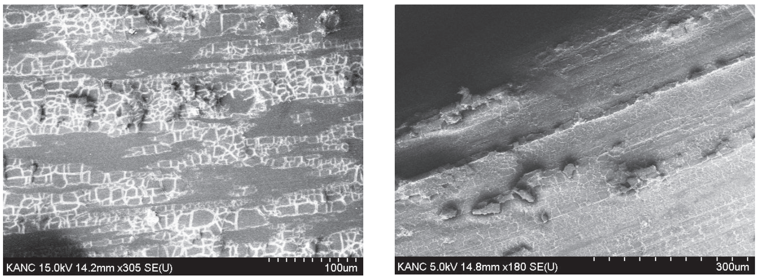

Figure 1: Rectangular plate resting on elastic foundations.

2. Methods

As shown in Fig. 1, five types of springs are used to describe the boundary conditions. K_w and K_s are linear Winkler foundation and linear Pasternak foundation, respectively. The Lagrangian function is:

$$L = T - U_f - V_{plate} - V_{spring} \tag{1}$$

According to Refs. [4, 5], we can easily get the total kinetic energy (T) and strain energy (V_{plate}) and $V_{spring}^{uniform}$. For the case considered, the total energy V_{spring}^{points}, due to multi-points supports, and the strain energy U_f , due to the Pasternak foundations, are given by:

$$\begin{aligned} V_{springs}^{points} = & \frac{1}{2}\sum_{r=0}^{NR}\left[Q(0,y_r)^T K_{x0} Q(0,y_r) + Q(a,y_r)^T K_{xa} Q(a,y_r)\right] \\ & + \frac{1}{2}\sum_{s=0}^{NS}\left[Q(x_s,0)^T K_{y0} Q(x_s,0) + Q(x_s,b)^T K_{yb} Q(x_s,b)\right] \end{aligned} \tag{2}$$

$$U_f = \frac{1}{2}\int_A \left\{ K_w w^2 + K_s\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]\right\} dA \tag{3}$$

where K_{ij} and Q are springs matrix and displacements matrix, according to Refs. [5, 6]. Finally, we arrive at the following matrix equation, from which the natural frequencies can be obtained by solving a standard matrix Eigen problem:

$$(K - \omega^2 M)G = 0 \tag{4}$$

3. Results

The first four frequency parameters $\Omega=(\omega a^2)[\rho/E_2 h^2]^{1/2}$ for the isotropic laminated plates with different boundary constraints presented in the Table 1 agree well with the Finite Element Method (FEM) numerical results. In Figs. 2 and 3, the variations of the frequency parameters Ω against the number of layers n and points q are depicted. When n=10, the frequency parameters may reach their crest and then remain unchanged. When the total numbers of the clamped points reaches 28, the frequency parameters become stable, which can effectively approximate the uniformly clamped boundary condition.

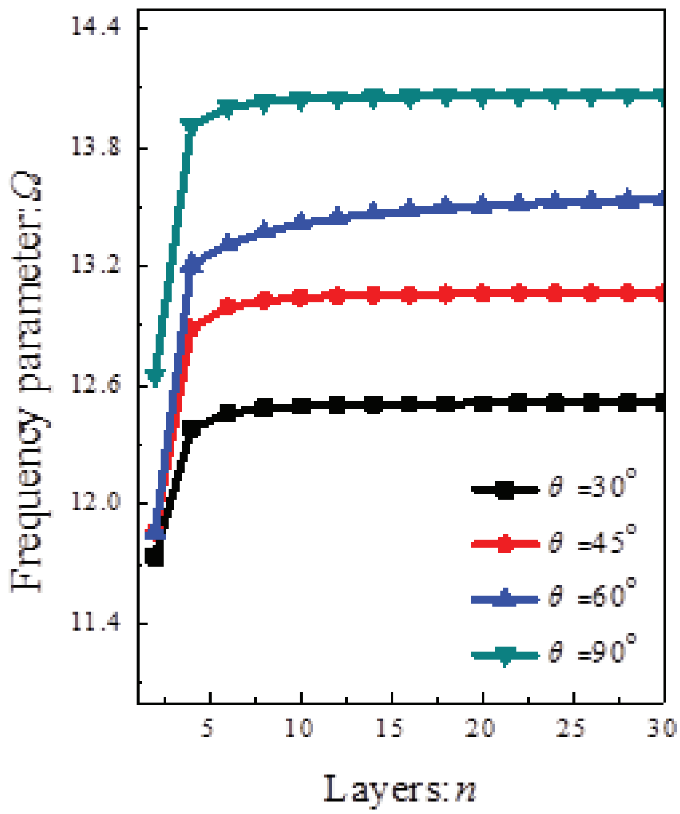

Figure 2: Dependence of frequency parameter Ω on layers number n for plate.

BC	Methods	Model sequence			
		1	2	3	4
Free	Present	3.6627	5.4012	6.5182	8.4358
	FEM	3.5986	5.3627	6.4682	8.3880
Uniform elastic	Present	3.6438	5.3827	6.5191	8.4369
	FEM	3.6012	5.3162	6.4705	8.3909
8-points elastic	Present	3.6499	5.3898	6.5248	8.4439
	FEM	3.6012	5.3262	6.4705	8.3909

Table 1: Frequency parameter Ω for plate with different BC.

4. Conclusion

In this paper, we can easily find that multi-points supports can effectively approximate the uniformly clamped boundary conditions, which is the basic idea of grid division in FEM.

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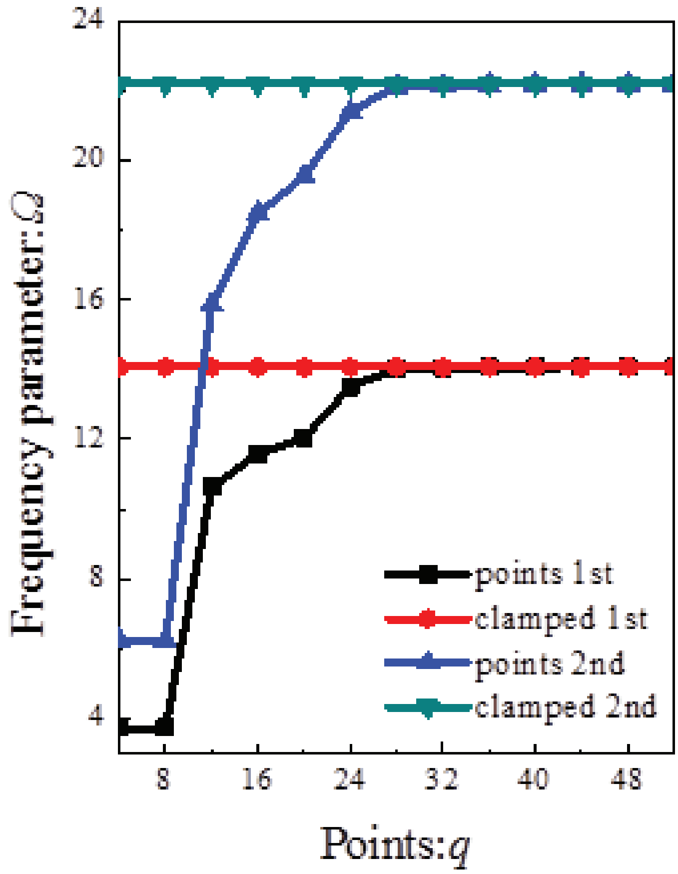

Figure 3: Dependence of frequency parameter Ω on points number q for plate.

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